ROYAL
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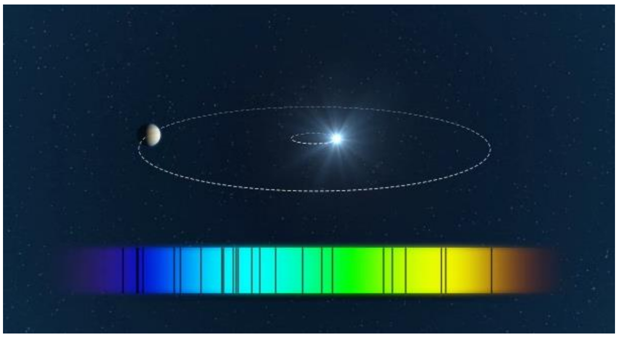
Finding the Mass of an Exoplanet from Radial Velocity

Post-16

Topics covered: extrasolar planets, Doppler effect, radial velocity curves, angular momentum, astronomical units

Watch "Alien worlds and the Doppler effect".
https://vimeo.com/64551416

The vast majority of stars in the Milky Way have planets orbiting them, these are called exoplanets. Their gravitational effects result in a small wobble in the star. This can be measured by analysing the light in the form of a spectrum. Elements in the star's outer atmosphere absorb specific frequencies of light leaving behind a pattern of dark lines in the star's spectrum. These absorption lines shift periodically to higher (bluer) and lower (redder) frequencies as the star wobbles towards and away from us.



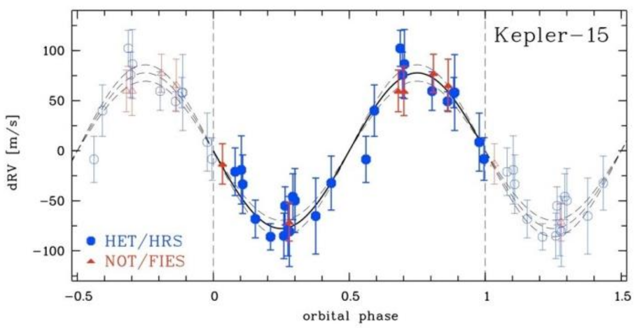
Credit: ESO/L. Calçada

The velocity of the wobble can be calculated using the Doppler equation:

$$\frac{v}{c} = \frac{\lambda - \lambda_0}{\lambda_0}$$

Where v = velocity of wobble, c = speed of light (3×10^8 m/s), λ = observed wavelength of line, λ_0 = stationary wavelength.

1. The star Kepler 15 has an orbiting planet, Kepler 15b, causing it to wobble. Calculate the velocity of its wobble, V_*, if the observed wavelength of its hydrogen alpha (Hα) absorption line is $6563.0017 \times 10^{-10}$ m and the stationary wavelength of Hα is 6563×10^{-10} m.



Credit: NASA/Kepler

A radial velocity curve shows the change in velocity of the wobbling star along our line of sight. It looks like a sine wave and shows when the star is wobbling towards and away from us.

The radial velocity curve of Kepler 15 can be seen in the diagram on the previous page. The mass of the star, M_*, is 2.02×10^{30} kg (1.02 x mass of the Sun).

Angular momentum must be conserved in any system:

$$M_* V_* r = m_p v_p r$$

Where m_p is the mass of the planet and v_p is the velocity and r is the radius of the planetary orbit which cancels out. v_p can be calculated from its orbital period, T (in seconds) and its orbital radius, a_p (in metres):

$$v_p = \frac{2\pi a_p}{T}$$ **(units of m/s)**

2. Find the mass of Kepler 15b if its orbital period is 4.943 days and the semi major axis of its (elliptical) orbit is 0.05714 AU, where 1 AU = 1.5×10^{11} metres.

3. How does the mass of Kepler 15b compare to Jupiter (mass = 1.898×10^{27} kg)?

Finding the Mass of an Exoplanet from Radial Velocity: ANSWERS

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Finding the velocity of the star Kepler 15:

$V_* = c \times (\lambda - \lambda_0)/\lambda_0$

$V_* = 3 \times 10^{8} \times (6563.0017 \times 10^{-10} - 6563 \times 10^{-10}) / 6563 \times 10^{-10}$

$V_* = 78$ m/s.

Finding the mass of the exoplanet Kepler 15b:

$M_* V_* = 2.02 \times 10^{30} \times 78 = 1.5756 \times 10^{32}$ kg m/s

$v_p = (2 \times \pi \times 0.05714 \times 1.5 \times 10^{11})/(4.943 \times 24 \times 3600) = 1.26 \times 10^{5}$ m/s

so

$m_p = M_* V_* / v_p = 1.25 \times 10^{27}$ kg = **0.66 x mass of Jupiter**