Age of the Universe
Post-16

Topics covered: galactic spectra, Doppler effect, Hubble's parameter, linear regression, error analysis, cosmological units

Teacher's Notes

This activity looks at Hubble's law ($v = H_0 d$) whereby students use real data from the Sloan Digital Sky Survey to plot a graph from which they can obtain the Hubble parameter, H_0. Students then look at the possible sources of error in their data and use this to calculate the uncertainty in their value for H_0.

Equipment: computer with MS Excel software, calculator, "Table of galaxies" excel document

Questions to ask the class before the activity:

How do we know the Universe is expanding?
Answer: we look at the light from galaxies and we know they are moving away from us and each other, accelerating with distance. Edwin Hubble first discovered this in 1923 using his 100" reflector on Mount Wilson in the US.

What problems or constraints might there be in trying to measure the motion of galaxies?
Answer: very distant galaxies are faint and difficult to measure; the telescope must have a high sensitivity but also be capable of resolving emission spectra from galaxies. The atmosphere affects observations and so a space telescope is useful for these kinds of studies but they can't be as big as terrestrial telescopes.

How would we measure the distance to a galaxy and its velocity?
Answer: measure its intensity and find its luminosity to get the distance and use its spectrum to find the velocity from Doppler shifted lines.

Questions to ask the class after the activity:

What observational constraint related to galaxies would put an absolute lower limit on the age of the Universe?
Answer: the most distant galaxy observed must be younger than the age of the Universe i.e. the oldest galaxy seen so far (by the Hubble Space Telescope) is 13 billion light years away, this is 0.7 billion years less than the current value for the age of the Universe.

If the Hubble parameter was greater or smaller than the current value of 70.1, what would that tell you about the Universe?
Answer: a larger value would give a younger age for the Universe. A steeper gradient of the velocity-distance graph would mean galaxies have greater velocities at a certain distance, this could suggest a larger acceleration in the expansion of the Universe which may mean a lower density of matter or a greater repulsive force from something like dark energy.

Calculating the Error in H_0: Answers

Use the following equations to find the velocity of your galaxy if the measured wavelength, λ of the Hα line = 7094 ± 1 Å and its rest wavelength, λ_0 of the Hα line = 6562.790 ± 0.001 Å. Is this a blueshift or a redshift?
Answer: redshift (observed wavelength is longer than rest wavelength)

Find the error, Δv in your calculated velocity, v using the equations below:

$$\frac{v}{c} = \frac{\lambda - \lambda_0}{\lambda_0} = z \qquad \left(\frac{\Delta v}{v}\right)^2 = \left(\left(\frac{\Delta \lambda}{\lambda}\right)^2 + \left(\frac{\Delta \lambda_0}{\lambda_0}\right)^2\right)$$
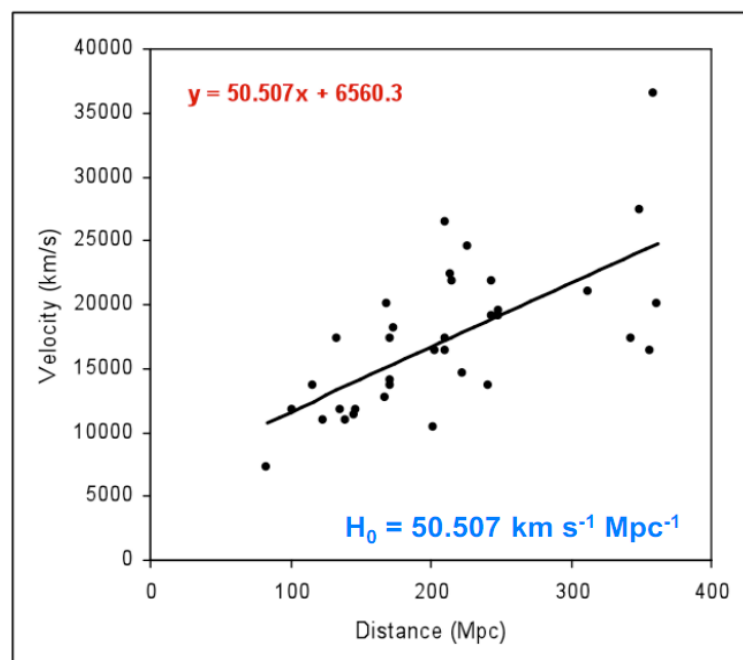
Recessional velocity, v = 24 547 km s^{-1}

Error, Δv = 3.46 km s^{-1}

Percentage error = 0.014%

c = 299 792 458 m s^{-1}

Calculate the value of H_0 for each row in your excel document "Table of galaxies" (velocity ÷ distance) and take the average of these values. Write this down at the bottom of your table.
Answer: 85.218

$y = 50.507x + 6560.3$

H_0 = 50.507 km s^{-1} Mpc^{-1}

Compare this number to the gradient of your graph. Why are they different? Which method gives a more accurate value for H_0?

Answer: The two values are different because one was calculated using values of H_0 from each of the galaxies and the other was taken from a best fit line through the points. This method is more accurate as it uses the whole sample to get a truer estimate of H_0 and not individual galaxies where the uncertainty is much larger.

What is the error in your measured value for H_0? Is it what you expected?

Answer: 0.0082. The value of H_0 is 50.507 ± 0.008 km s^{-1} Mpc^{-1}. This is a very small error - looking at the graph and the scatter of points around the line a much larger error in H_0 is expected. We did not take into account the error in the distance along with the assumptions about luminosity.

Activity: Age of the Universe

Hubble's law states that the **recessional velocity** of galaxies increases with their **distance** from us. This is a linear relationship and is represented by the formula **$v = H_0 d$**. The gradient of this straight line is defined as the Hubble parameter, H_0, which is has the units km s^{-1} Mpc^{-1}, where Mpc is megaparsec, an astronomical unit equivalent to 3.09×10^{19} km.

The **distance** of a galaxy can be determined from its measured intensity (light arriving at a telescope) and its intrinsic luminosity (total light emitted from galaxy). The **velocity** of the galaxy can be calculated from its emission spectrum. A galaxy spectrum often shows strong hydrogen emission lines such as Hα (below). By measuring the wavelength of this observed line and comparing it to the rest wavelength we can tell whether it is redshifted or blueshifted and then we can calculate the velocity of the galaxy using the Doppler equation.

Galaxy 1

Luminosity (W)	Intensity (W m^{-2})	Wavelength Hα (Å)
1×10^{37}	16.26×10^{-15}	7094
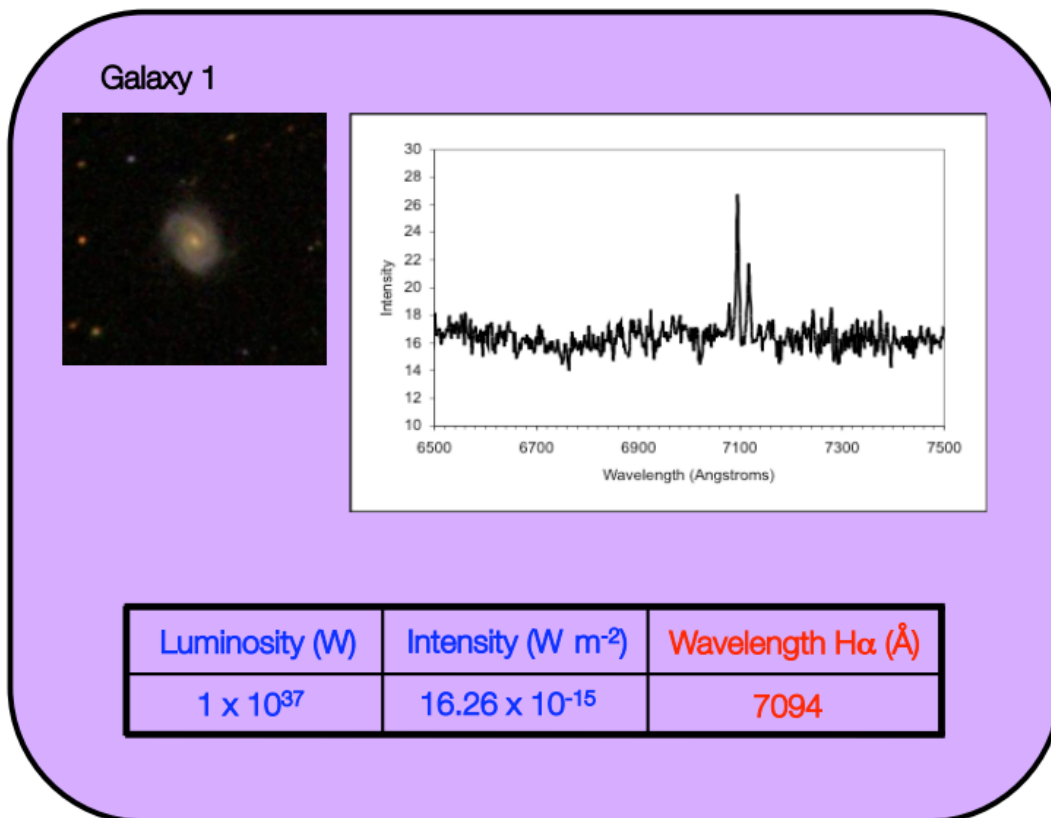
Every measurement or observation has an uncertainty or **error**. The measured wavelength of Hα from the galaxy spectrum will have an uncertainty that is dependent on the resolution of the instrument used. For example if the measured wavelength is 7094 Å, where 1 Å (angstrom) = 0.1 nm, the error in this value, Δλ = 1 Å. There will also be an error in the

laboratory wavelength which would be very small but here we will make it a bit bigger: rest wavelength of Hα = 6562.790 Å, error, $\Delta\lambda_0$ = 0.001 Å.
The distance of the galaxy can be calculated from its luminosity (Watts) and its intensity (Watts per metre squared). A typical measured value for the intensity of the galaxy might be 16.26 x 10^{-15} W m^{-2}. The associated error, ΔI, for a value of 2 dp accuracy would be 0.01 x 10^{-15} W m^{-2}.
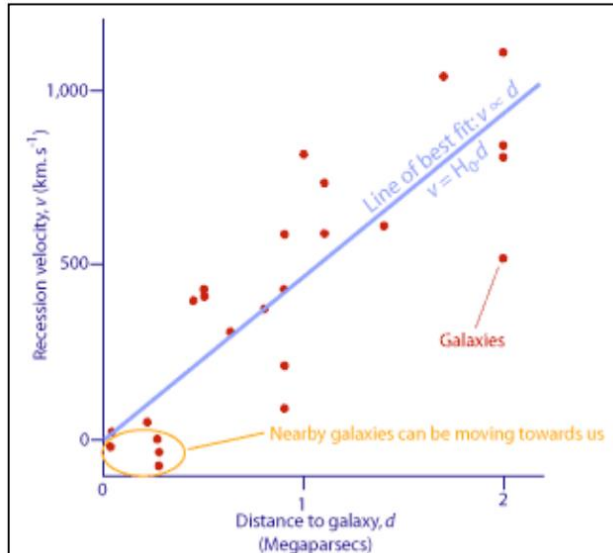
In this exercise we will be focusing on the **error in the velocity only** and using that to calculate an error for the Hubble parameter, **H$_0$**.

We can obtain a more accurate value of H$_0$ by using a large sample of galaxies and plotting their velocities against their distances in a graph (see left). A value for H$_0$ can be determined from the gradient of the best fit line.

Calculating the Error in H$_0$

The quantity we wish to determine is derived from several measured quantities. Errors in measured values can be incorporated into functions to determine the error in the final calculated value. Consider two in dependent measured quantities X and Y and their errors ΔX and ΔY. The error in the derived quantity, ΔF can be determined as follows:

1) $F = X + Y$ or $F = X - Y$ $\qquad \Delta F^2 = \Delta X^2 + \Delta Y^2$

2) $F = XY$ or $F = X/Y$ $\qquad \left(\dfrac{\Delta F}{F}\right)^2 = \left(\dfrac{\Delta X}{X}\right)^2 + \left(\dfrac{\Delta Y}{Y}\right)^2$

Use the following equations to find the velocity of your galaxy if the measured wavelength, λ of the Hα line = 7094 ± 1 Å and the rest wavelength, λ_0 of the Hα line = 6562.790 ± 0.001 Å. Is this a blueshift or a redshift?

Find the error, Δv in your calculated velocity, v using the equations below (the speed of light, c = 299 792 458 m s⁻¹ or **299 792.458 km s⁻¹** :

$$\frac{v}{c} = \frac{\lambda - \lambda_0}{\lambda_0} = Z \qquad \left(\frac{\Delta v}{v}\right)^2 = \left(\left(\frac{\Delta \lambda}{\lambda}\right)^2 + \left(\frac{\Delta \lambda_0}{\lambda_0}\right)^2\right)$$

Recessional velocity, v = _____ km s⁻¹

Error, Δv = _____ km s⁻¹

c = 299 792 458 m s⁻¹

Percentage error = _____

Plot a velocity-distance graph in Excel:

Open up the 'Table of Galaxies' excel file. Select the distance and velocity columns of your table (just the values - see below). Click on the bar chart symbol on the toolbar and select 'scatter' for type of graph. Follow the instructions.

Galaxy	Constellation	Distance (Mpc)	Velocity (km s⁻¹)	H_0 (km s⁻¹ Mpc⁻¹)
1	Hercules	83.925	7176.596	
2	Perseus	226.695	24546.701	
3	Sextans	136.544	11747.676	
4	Musca	172.316	13576.108	
5	Pegasus	359.465	36431.510	

Add a best fit line:

Your graph should look like the one on the right. Right click on the data points and select 'Add Trend line', select 'linear' and then click on 'Options' and select 'Display equation on chart'. The gradient of the graph gives Hubble's parameter, H_0. Write down your value for H_0 on your graph with the correct units.
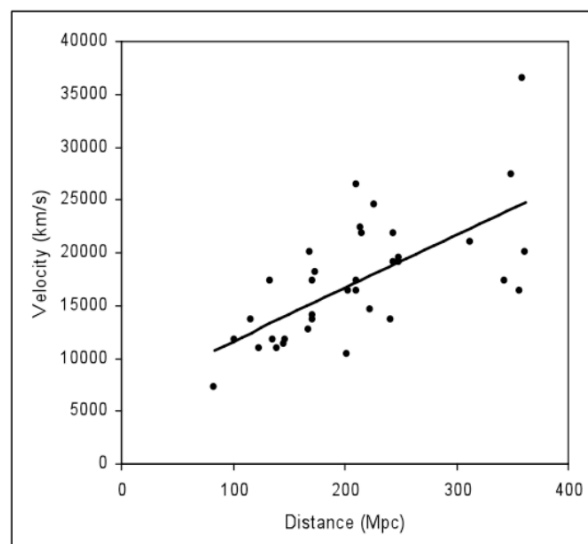
Calculate the value of H_0 for each row in your table (velocity ÷ distance) and take the average of these values. Write this down at the bottom of your table. Compare this number to the gradient of your graph. Why are they different? Which method gives a more accurate value for H_0?

Use the equations (below) to calculate the error in the gradient of your graph, σ_m. You will need to use your calculated error in your velocity, Δv. Use the table (below) in the excel file to help you use the equations.

$$\sigma_m^2 = \frac{\sigma^2}{N\left(\overline{x^2} - \overline{x}^2\right)} \qquad \overline{x} = \frac{\sum x}{N} \qquad \overline{x^2} = \frac{\sum x^2}{N}$$

σ_m = Error in gradient
σ = Error in y value (Δv)
x = x value (distance)
N = number of values
Σ = Sum of all the numbers, 1...N

x	x^2
$\downarrow$	$\downarrow$
Σx	Σx^2

What is the error in your measured value for H_0? Is it what you expected?

Calculating the age of the Universe

The units of H_0 are km s^{-1} Mpc^{-1}. 1 megaparsec = 3.09 x 10^{19} km. The value of H_0 gives the age of the Universe. Calculate the age in seconds and then convert this into years.

Latest value for H_0

The most accurate value for the Hubble constant has been determined using a different technique to Hubble's law. The ESA Planck satellite has measured temperature fluctuations as small as 1 microKelvin in the left-over radiation from the Big Bang (called the cosmic microwave background or CMB). Ripples in the CMB reveal the initial conditions for the formation of galaxies and allow scientists to determine the shape and fate of the Universe.

The current value for the Hubble parameter is **67.4 ± 1.4 km s^{-1} Mpc^{-1}**. This gives a value for the age of the Universe, **13.81 ± 0.06 billion years** (approximately 3 times the age of the Solar System).