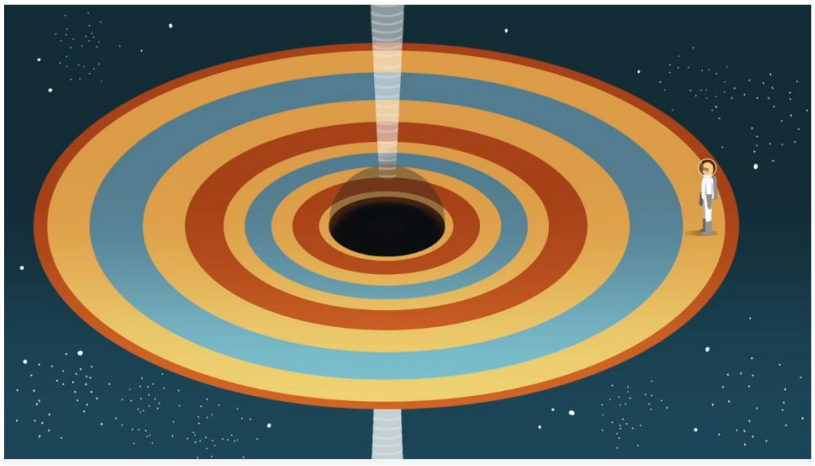
ROYAL OBSERVATORY GREENWICH

Calculating the size of a black hole

Post-16

Topics covered: Gravity, black holes, Schwarzschild radius, light-years

Watch the video "What's inside a black hole?" https://vimeo.com/88896853



Black holes exist in a broad range of sizes and masses, from stellar mass black holes (5-10 solar masses) to supermassive black holes (millions or billions of solar masses).

The Schwarzschild radius indicates the size of a black hole: if an object was placed at a distance equal to the Schwarzschild radius it would have to move at the speed of light to escape the intense gravitational field.

$$r_s = \frac{2Gm}{c^2} \qquad (1)$$

where r_s is the Schwarzschild radius (in metres), G is the gravitational constant = 6.67 x 10^{-11} m^3 kg^{-1} s^{-1}, m is the mass of the black hole (kg) and c is the speed of light = 3 x 10^8 m s^{-1}.

1. By how much must the Earth shrink to become a black hole? The (equatorial) radius of the Earth is 6378 km and its average density is 5.5 g cm^{-3}. Make sure your units are consistent.

2. The mass of the Milky Way is 1250 billion solar masses and its diameter is 100 000 light-years. A light-year is the distance that light travels in a year e.g. if a star is 0.2 light-years away the light has travelled for 0.2 years. The mass of the Sun is 1.989 x 10^{30} kg.
   Calculate the size our galaxy must be to become a black hole in light-years.

ROYAL OBSERVATORY GREENWICH

Calculating the size of a black hole: ANSWERS

Post-16

1. r_s of Earth = 9 mm, diameter would have to shrink by a factor of 7.1×10^8 (710 million)
2. 3.7×10^{15} metres = 0.4 ly