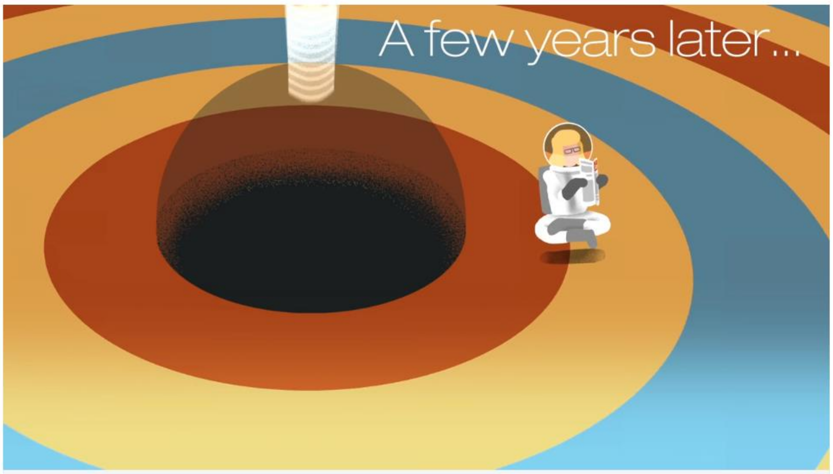
Time dilation by a black hole

Post-16

Topics covered: Gravitational time dilation, black holes, Schwarzschild radius, light-years

Watch the video "What's inside a black hole?"
https://vimeo.com/88896853



Time dilation occurs close to a black hole, this means that to an observer far away from the black hole time near the black hole runs slower. The extent of time dilation can be determined from the size of the black hole, r_s and the distance of the observer, r:

$$\frac{t_r}{t} = \sqrt{1 - \frac{r_s}{r}} \qquad (1)$$

In equation (1) t is the time elapsed for an observer outside the influence of the black hole and t_r is the time elapsed for an observer at distance r.

The Schwarzschild radius indicates the size of a black hole:

$$r_s = \frac{2Gm}{c^2} \qquad (2)$$

where r_s is the Schwarzschild radius (in metres), G is the gravitational constant = 6.67 x 10^{-11} m^3 kg^{-1} s^{-1}, m is the mass of the black hole (kg) and c is the speed of light = 3 x 10^8 m s^{-1}.

1. Using equation (2) find the Schwarzschild radius, r_s of the black hole in the centre of the Milky Way. The mass of the black hole is 4 million solar masses, where the mass of the Sun is 1.989×10^{30} kg.

2. Convert r_s into light-years (this will be a very small number). A light year is the distance light travels in a year e.g. if a star is 0.2 light-years away the light has travelled for 0.2 years.

3. Using equation (1) find the time elapsed for an observer at distance r from the black hole, t_r if the time elapsed for Brendan on a planet in another galaxy far away from the Milky Way, t, is 10 years. Do this for (i) $r = 2r_s$; (ii) $r = 5r_s$; (iii) $r = 10r_s$ and (iv) $r = 26000$ ly (this is the distance of the Earth from the centre of the Milky Way).

ROYAL OBSERVATORY GREENWICH

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1. $r_s = 1.18 \times 10^{10}$ metres
2. $r_s = 1.25 \times 10^{-6}$ ly
3. (i) $r = 2r_s$; $t = 7.1$ years
   (ii) $r = 5r_s$; $t = 8.9$ years
   (iii) $r = 10r_s$; $t = 9.5$ years
   (iv) $r = 26000$ ly; $t = 10$ years